

Optimization problems, Knapsack

6.1000 LECTURE 20

FALL 2025

Announcements

- Midterm 2 results
 - Regrades open until tomorrow 6 pm
 - Drop date is Wednesday 11/19
 - Grading clarifications
 - Exam scores are not curved
 - Final letter grade is not solely based on total score
- Pset 6 released today
- Pset 7 released next Monday

Remainder of term

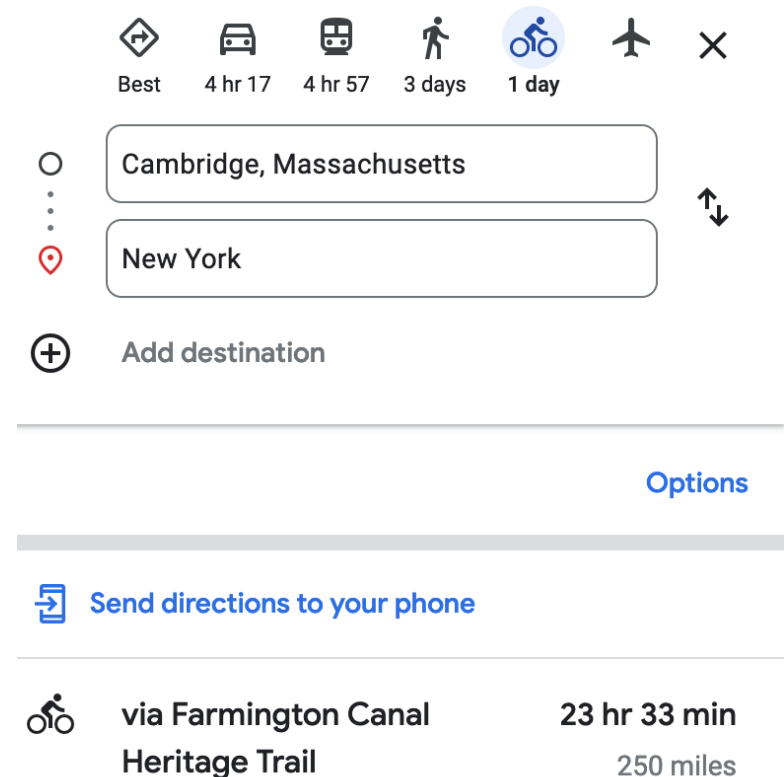
- No new Python
- This week
 - Generalizing shortest paths to other optimization problems
- Next two weeks
 - Building models from data, making statistical conclusions

Review Midterm 2 Question 3

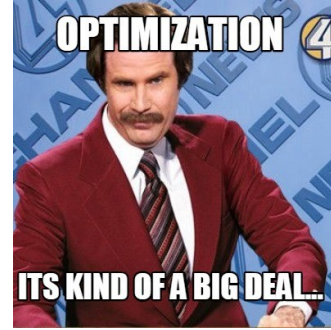
- Three big ideas
 1. Using class attribute **count** to set instance attribute **id**
 2. Manage attributes within class, even with inheritance
 - Syntax of inheritance and **super()**
 3. Writing dunder method **__mul__()**
 - First arg: **Virus** object
 - Second arg: Cell/LungCell object

Optimization problems

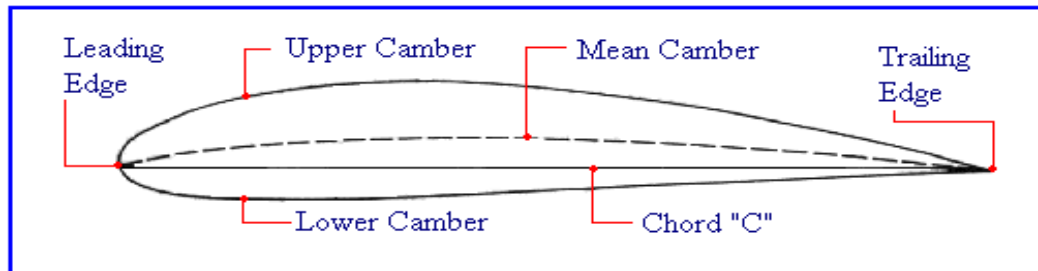
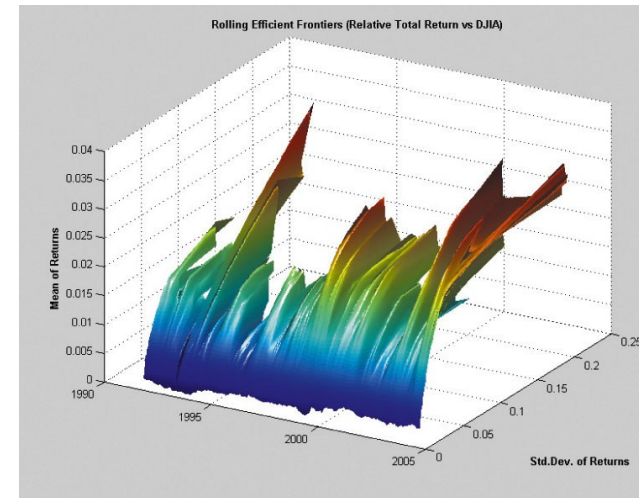
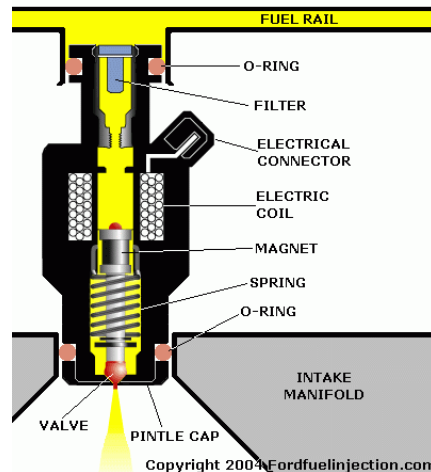
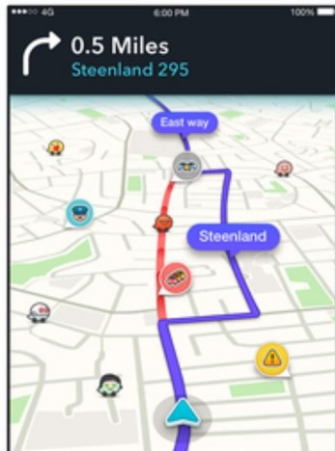
- An **objective function** that is to be maximized or minimized
 - E.g., minimize money spent traveling from Cambridge to NYC
- But perhaps merely optimizing objective functions is not enough
- A **set of constraints** (possibly empty) that must be honored
 - E.g., expected transit time < 5 hours



Optimization problems

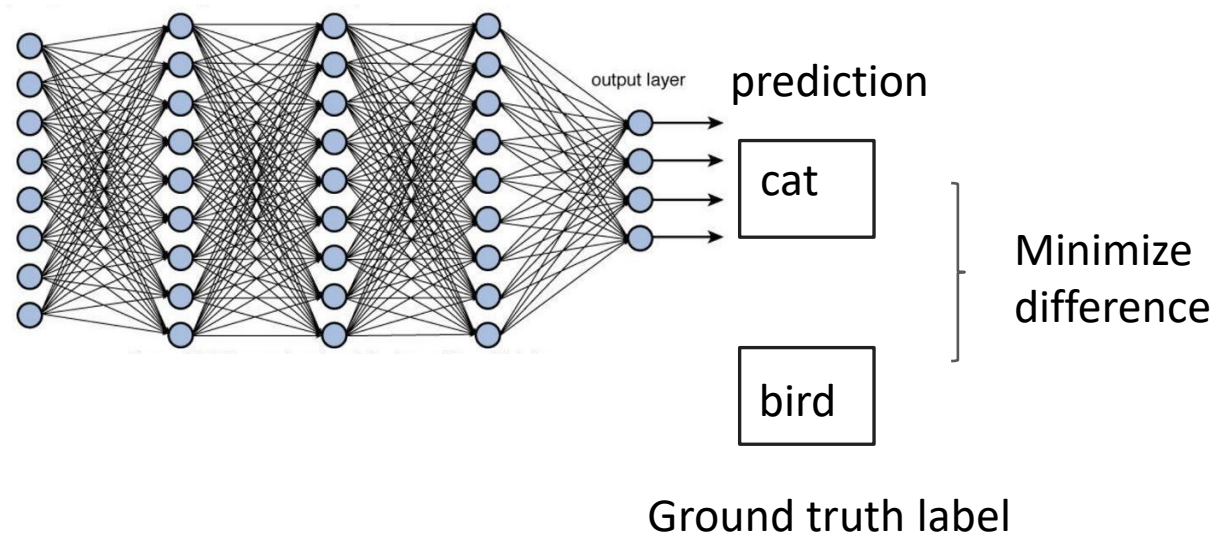


- Anytime you are trying to maximize or minimize something, you are solving an optimization problem



Deep learning / generative AI

- Trained by **minimizing** a loss function
 - E.g. minimize difference between prediction and ground truth training data



Graph optimization problems

- Shortest paths
 - Task: find a path
 - Objective: minimize path weight
 - Constraints: valid path, start, destination
 - Applications: routing, scheduling
- Minimum spanning tree
 - Task: find a tree subgraph
 - Objective: minimize weight of tree edges
 - Constraints: selected edges form a tree, reach every node
 - Applications: road network coverage, power delivery
- Maximum flow / minimum cut
 - Task: “transport” a quantity of product across a network

Other common optimization problems

- Knapsack
 - Task: fill a bag with subset of available items
 - Objective: maximize total value
 - Constraints: stay under bag's weight limit
 - Applications: packing delivery trucks, managing power allocation, election flipping
- Bin packing
 - Task: minimize number of containers needed to ship items
- Edit distance
 - Task: change one sequence of letters into another sequence
 - Objective: minimize total cost of edit actions
 - Constraints: available actions (insert, delete, substitute)
 - Applications: tracking DNA mutations, file compression and error correction

Knapsack Problems



Also known as a rucksack, haversack, backpack

- You have limited strength, so there is a maximum weight knapsack that you can carry
- You want to take more stuff than you can carry
- How do you choose what to take vs. leave behind?
 - Want to optimize “value” of things to take
- Two variants
 - Continuous or fractional knapsack problem
 - 0/1 knapsack problem

Straightforward

Much more interesting

Quanta are infinitesimal relative to available space



versus



Quanta are large relative to available space

A Fun 0/1 Knapsack Problem



Pos	Player	Points	Salary
QB	Patrick Mahomes @ JAX (9)	22.7	\$ 7200
RB	Leonard Fournette vs KC (31)	19.7	\$ 6100
RB	Dalvin Cook vs ATL (28)	18.8	\$ 6000
WR	Curtis Samuel vs LAR (22)	12.2	\$ 4200
WR	Tyler Lockett vs CIN (10)	15.6	\$ 6000
WR	Marvin Jones @ ARI (13)	13.8	\$ 4800
TE	Zach Ertz vs WAS (4)	16.9	\$ 6100
Flex	Kerryon Johnson @ ARI (27)	18.1	\$ 5800
DEF	Baltimore Ravens @ MIA (24)	10.4	\$ 3800
Total		148.2	\$50000

Objective function: Maximize total “score” for roster

Constraints:

Total cost $\leq$ \$50k

1 QB, 2 RB, 3 WR, 1 TE, 1 Flex, 1 Team Defense

Example: Participatory Budgeting

- Cambridge has had 9 annual cycles of participatory budgeting
 - <https://pb.cambridgema.gov/pbcycles>
- Set aside about \$1 million
- Release proposed list of projects, each with a cost
- Collect votes from residents, score each project on collective benefit

Winning PB projects	Planning/Design	Procurement of Supplies/ Equipment	Installation
Trees for Danehy Park and Cambridge	✓	Locations for plantings have been identified. DPW is waiting until digging can take place at the park.	
Technology for Youth Centers	Human Services is planning the distribution process for technology in city youth centers.		
Look and Listen: Safer Crossing for Cambridge	The Department of Transportation is planning for installation and identifying locations for safer crosswalks.		
Smart Recycling and Trash Compactors	✓	Public Works is in the process of ordering the new recycling and trash compactor units.	
Electric Vehicle Charging Stations	The City is identifying locations with highest demand and planning for installation of EV charging stations.		
E-Cargo Bikes for Watering Trees	✓	✓	✓
More Outdoor Public Wi-fi	The IT Department is planning for installation of necessary wifi in Clement Morgan Park.		
Three New Public Art Murals	✓	✓	✓
	Planning for the mural in the Port is underway Planning for the mural in Central Square is underway		

YOUR TURN

- Is it a knapsack problem?
 - Picking stocks for a portfolio
 - Ranking your professors
 - Deciding which subjects to take freshmen year
 - Finding the shortest path between two places
 - Debugging a Python program
 - Managing work/life balance

